647510 **BSc4MatC0401** **Seat No: ___________**

B.Sc. Semester - 4 (CBCS) Examination
March/April- 2018
MATHEMATICS
(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) ANSWER THE FOLLOWINGS: 04
(1) Define: Subspace
(2) Define: Spanning Set
(3) A set of vectors containing the zero vector is always a linearly dependent set in a vector space is True/ False?
(4) If W is a subspace of a vector space V, then Sp(W)= _____

Que-1(B) ANSWER ANY ONE: 02
(1) Define: Linearly Independent Set of vectors.
(2) If (a, b, c), (d, e, f) and (g, h, i) are linearly dependent vectors of the vector space R^3, then the matrix $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ is singular, True or False ?

Que-1(C) ANSWER ANY ONE: 03
(1) Show that W={(a,b,c)/2a + 3b - c = 0} is a subspace of R^3.
(2) Check whether the set A={e^x,e^{2x}.e^{3x}) of the vector space $\mathcal{C}[0,2\pi]$ is a linearly independent set or not.

Que-1(D) ANSWER ANY ONE: 05
(1) If A be any non-empty set of a vector space V, then prove that Sp(Sp(A)) = Sp(A).
(2) Show that V={(a,b) $\in R^2$ /b>0} is a vector space w.r.t. the following addition and scalar multiplication.
$\forall$ (a,b), (c,d) $\in R^2$ and $\forall\ \alpha \in R$,
(a,b) + (c,d) = (a + c, bd) and α(a,b) = (αa, b^{α}).

Que- 2(A) ANSWER THE FOLLOWINGS: 04
(1) Define: Basis of a vector space
(2) A vector space can have many basis – True or False?
(3) If W is a subspace of a vector space V, then Dim(V) $\geq$ Dim(W) —True or False?
(4) Define: Complementary Subspaces

Que-2(B) ANSWER ANY ONE: 02
(1) Prove that B={(1,2,3), (4,5,6), (7,8,9)} is not a basis of the vector space .
(2) State extension theorem.

Que-2(C) ANSWER ANY ONE: 03
(1) Find coordinates of the vector (1,2,3) with respect to the basis B={$e_2 + e_3$, $e_3 + e_1$, $e_1 + e_2$}.
(2) Extend A={(1,0,-1,0), (2,3,0,-1)} to a basis of the vector space V=R^4.

Que-2(D) ANSWER ANY ONE: 05
(1) State and prove the existence theorem for a basis for a vector space.
(2) If W_1 and W_2 be any two subspaces of a vector space V, then prove that Dim($W_1 + W_2$) = Dim(W_1) + Dim(W_2) - Dim($W_1 \cap W_2$).

Que-3(A) ANSWER THE FOLLOWINGS: 04
(1) Define: Linear Transformation
(2) Define: Kernel of a linear transformation
(3) If T : U$\to$V be any linear transformation then n(T) $\geq$ Dim(U)-- -True or False?
(4) Define: Nilpotent linear transformation.

Que-3(B) ANSWER ANY ONE: 02
(1) Verify: T : $R^2 \to R^2$, T(a,b) = (a + 3,b + 1), $\forall$(a,b) $\in R^2$ is a linear transformation or not.
(2) Let T, S : $R^2 \to R^2$ be linear transformations such that T(e_1) = -e_2, T(e_2) = e_1;S(e_1) = 2e_2, S(e_2) = 7e_1 then find 2T + 3S.

Que-3(C) ANSWER ANY ONE: 03
(1) If T : U $\to$ V be a one-one linear transformation and if {V_1, V_2, ... V_n} be a linearly independent set

of U, then prove that the set $\{T(V_1), T(V_2), ...T(V_n)\}$ is a linearly independent set of V.
(2) If $T : V \to V$ be a linear transformation such that $T^2 = T + I$, then prove that T is non-singular.

Que-3(D) ANSWER ANY ONE: 05
(1) Prove that a linear transformation $T : U \to V$ is one-one iff $NT = \{\theta\}$.
(2) State and prove the rank-nullity theorem.

Que-4(A) ANSWER THE FOLLOWINGS: 04
(1) Define: Linear Functional
(2) Define: Dual of a vector space
(3) Define: Eigen Value of a linear transformation
(4) Define: T-eigen basis

Que-4(B) ANSWER ANY ONE: 02
(1) Define: Adjoint of a linear transformation
(2) If $T : V \to V$ be a linear transformation and B be a basis of V, then what will be the value of $\det([T - \lambda I_n), B])$ if λ is an eigen value of T?

Que-4(C) ANSWER ANY ONE: 03
(1) Explain: Matrix associated with a linear transformation
(2) Let $T : R^2 \to R^2$, $T(a,b) = (a-b)$, $\forall (a,b) \in R^2$ and let $B_1 = \{e_1,e_2\}$, $B_2 = \{e_1 + e_2, e_1-e_2\}$ then find $[T;B_1,B_2]$.

Que-4(D) ANSWER ANY ONE: 05
(1) If $T : V \to V$ be a linear transformation and B be any basis of V, then prove that T is singular $\Leftrightarrow \det([T;B]) = 0$.
(2) Find eigen values and eigen vectors for the linear transformation
$T : R^3 \to R^3$,
$T(a,b,c) = (-2b -2c, -2a -3b -2c, 3a + 6b + 5c)$, $\forall (a,b,c) \in R^3$ by considering the standard basis of R^3 .

Que-5(A) ANSWER THE FOLLOWINGS: 04
(1) Define: Tangent to a curve
(2) Define: Radius of curvature
(3) Define: Point of inflexion
(4) Define: Asymptote

Que-5(B) ANSWER ANY ONE: 02
(1) Define: Curvature
(2) Show that the curve y=exp(x) is concave upward everywhere.

Que-5(C) ANSWER ANY ONE: 03
(1) Find the radius of curvature at any point P(x,y) on the curve $y^2 = 4ax$.
(2) Find asymptotes parallel to the coordinate axis for the curve $x^2y^3 + x^3y^2 = x^3 + y^3$.

Que-5(D) ANSWER ANY ONE: 05
(1) If ρ be the radius of curvature for the polar curve $r = f(\theta)$, then prove that $= \frac{(r^2+2{r_1}^2)^{3/2}}{(r^2-rr_2+2{r_1}^2)}$
where $r_1 = \frac{dr}{d\theta}$, $r_2 = \frac{d^2r}{d\theta^2}$
(2) If $y = mx + c$ is an asymptote to the curve $y = f(x)$, then prove that
$m = \lim_{x\to\pm\infty} \frac{y}{x}$ and $c = \lim_{x\to\ \infty} (y - mx)$.

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